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PHASE-ONLY SYNTHESIS OF LINEAR AND PLANAR ARRAYS USING AUTOCORRELATION MATCHING METHOD

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Abstract

We present autocorrelation matching method as a phase-only synthesis method to design power pattern of both linear and planar antenna arrays. Equating the autocorrelation coefficients of an array having a presumed amplitude of antennas to those of a previously designed amplitude-phase array forms the basis of this method. Considering certain examples, the effectiveness of the proposed method for both linear and planar arrays has been verified.

I. Introduction

Antenna arrays are usually synthesized by amplitude-phase methods in which both amplitudes and phases of the antennas are changed [1-4]. Changing the phase of antennas of an array is more popular than changing their amplitudes. Indeed, in some arrays such as reflectarray, only phases of the unit cells being under control [5-7], we synthesize many linear and planar arrays by changing only the phase of their antennas. Such a procedure is called *phase-only synthesis* [8-13].

Almost all proposed methods for phase-only synthesis are based on optimization methods in two main categories. These include the evolutionary algorithms; like Genetic Algorithm (GA) [8], particle swarm optimization [9, 10], semidefinite relaxation technique [11], and the local search algorithms such as alternating projections method [12, 13].

Keywords and phrases: antenna array synthesis, autocorrelation matching method, phase-only synthesis.
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We present a different method for phase-only synthesis of both linear and planar antenna arrays, called *Autocorrelation Matching Method* (*AMM*). In this method, first an amplitude-phase array is synthesized through conventional methods such as Fourier's series method [1, 2], sampling method [1, 2] and perturbation of zeros [3]. Then the phases of antennas are obtained for given amplitudes. The second step of AMM is performed by equating the autocorrelation coefficients of the phase-only array to those of previously synthesized amplitude-phase array. The performance of the proposed method is verified by synthesizing both linear and planar arrays.

II. Autocorrelation of Linear Arrays

A linear antenna array is composed of N identical antennas of uniform inter-distances d on the z axis. The excitation current of the nth antenna is $I_n = A_n \exp(j\varphi_n)$. The radiation pattern of linear arrays is given by

$$F(\psi) = \sum_{n=0}^{N-1} I_n \exp(jn\psi), \tag{1}$$

where $\psi = kd\cos(\theta)$ is a real variable in which $k = 2\pi/\lambda$ and λ is the wavelength in the free space.

The power pattern of linear arrays resulted from (1) is

$$|F(\psi)|^2 = F(\psi)F^*(\psi) = \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} I_n I_m^* \exp(j(n-m)\psi)$$

$$= \sum_{l=-(N-1)}^{N-1} R_l \exp(-jl\psi). \tag{2}$$

In (2), R_l, in which $-(N-1) \le l \le N-1$, is the lth autocorrelation coefficient of all currents I_n is defined as follows:

$$R_l = \sum_{n=-(N-1)}^{N-1} I_n I_{n+l}^* = \sum_{n=\max(0,-l)}^{\min(N-1-l,\, N-1)} A_n A_{n+l} \exp[j(\varphi_n - \varphi_{n+l})], \tag{3}$$

where the values of lower and upper bounds of the summation are due to the fact that the terms I_n and I_{n+l} exist.

It is seen from (2) that the power pattern, $|F(\psi)|^2$, is expressed as a Fourier series whose coefficients are the same as autocorrelation coefficients R_l. Since the power pattern $|F(\psi)|^2$

is real, $R_{-l} = R_l^*$. So, only the coefficients R_l, where $0 \le l \le N-1$ are independent out of $2N-1$ ones. In addition, if the power pattern $|F(\psi)|^2$ is symmetric or even, then the autocorrelation coefficients will be real and symmetric.

III. Autocorrelation of Planar Arrays

A rectangular planar antenna array consists of $M \times N$ antennas on the xy plane. The distances between the antennas along x and y directions are d_x and d_y, respectively. The excitation current of the mnth antenna is $I_{mn} = A_{mn}\exp(j\varphi_{mn})$. The radiation pattern of planar arrays is given by

$$F(\psi_x, \psi_y) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} I_{mn}\exp(j(m\psi_x + n\psi_y)), \tag{4}$$

where ψ_x and ψ_y are real variables defined as $\psi_x = 2\pi\frac{d_x}{\lambda}\sin\theta\cos\varphi$ and $\psi_y = 2\pi\frac{d_y}{\lambda}\sin\theta\sin\varphi$.

The power pattern of planar arrays as resulted from (1) is

$$\begin{aligned} |F(\psi_x, \psi_y)|^2 &= F(\psi_x, \psi_y)F^*(\psi_x, \psi_y) \\ &= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\sum_{m'=0}^{M-1}\sum_{n'=0}^{N-1} I_{mn}I^*_{m'n'}\exp(j(m-m')\psi_x + j(n-n')\psi_y) \\ &= \sum_{p=-(M-1)}^{M-1}\sum_{q=-(N-1)}^{N-1} R_{pq}\exp(-j(p\psi_x + q\psi_y)). \end{aligned} \tag{5}$$

In (5), R_{pq}, where $-(M-1) \le p \le M-1$ and $-(N-1) \le q \le N-1$, is the pqth autocorrelation coefficient of all currents I_{mn} defined as follows:

$$\begin{aligned} R_{pq} &= \sum_{m=-(M-1)}^{M-1}\sum_{n=-(N-1)}^{N-1} I_{mn}I^*_{(m+p)(n+q)} \\ &= \sum_{m=\max(0,-p)}^{\min(M-1-p,\, M-1)}\sum_{n=\max(0,-q)}^{\min(N-1-q,\, N-1)} A_{mn}A_{(m+p)(n+q)}\exp[j(\varphi_{mn} - \varphi_{(m+p)(n+q)})], \end{aligned} \tag{6}$$

where the values of lower and upper bounds of two summations are due to this fact that the terms I_{mn} and $I_{(m+p)(n+q)}$ exist.

From (4), it can be seen that the power pattern, $|F(\psi_x, \psi_y)|^2$, is expressed as a two dimensional Fourier series whose coefficients are the same as autocorrelation coefficients R_{pq}. Since the power pattern $|F(\psi_x, \psi_y)|^2$ is real, $R_{-p,-q} = R^*_{pq}$. So, only the coefficients R_{pq}, where $0 \le p \le M-1$ or $0 \le q \le N-1$, are independent out of $(2M-1)(2N-1)$ ones.

IV. Autocorrelation Matching Method

From two previous sections, it follows that the autocorrelation coefficients are the same Fourier's series coefficients of the power pattern. Hence, knowing the autocorrelation coefficients R_l or R_{pq} amounts to knowing the functions $|F(\psi)|^2$ or $|F(\psi_x, \psi_y)|^2$, respectively. Consequently, AMM is proposed for phase-only synthesis of linear or planar arrays to have a desired power pattern $|F(\psi)|^2$ or $|F(\psi_x, \psi_y)|^2$, respectively. In AMM, first a linear/planar array is synthesized using amplitude-phase methods, and then another linear/planar array, having specified amplitudes A_n or A_{mn}, is synthesized so that its autocorrelation coefficients are equated to those of the first synthesized array obtained by changing only the phase of the currents while the amplitude of the currents possess those specified values.

Consider the excitation currents of the phase-amplitude synthesized array given by $I'_n = A'_n \exp(j\varphi'_n)$ and $I'_{mn} = A'_{mn} \exp(j\varphi'_{mn})$ for linear and planar arrays, respectively. Also, $I_n = A_n \exp(j\varphi_n)$ and $I_{mn} = A_{mn} \exp(j\varphi_{mn})$ are the excitation currents of the phase-only synthesized linear and planar arrays, respectively. Equating the autocorrelation coefficients of phase-only synthesized arrays in (3) and (6) for $l = 0, 1, ..., N-1$ and $p = 0, 1, ..., M-1$ with those of phase-amplitude synthesized arrays, we obtain

$$\sum_{n=0}^{N-1-l} A_n A_{n+l} \exp(j(\varphi_n - \varphi_{n+l})) = R'_l, \tag{7}$$

$$\sum_{m=0}^{M-1} \sum_{n=\max(0,-q)}^{\min(N-1-q,\, N-1)} A_{mn} A_{(m+p)(n+q)} \exp[j(\varphi_{mn} - \varphi_{(m+p)(n+q)})] = R'_{pq}. \tag{8}$$

In (7) and (8), the autocorrelation coefficients of phase-amplitude arrays are defined as follows:

$$R'_l = \sum_{n=0}^{N-1-l} A'_n A'_{n+l} \exp(j(\varphi'_n - \varphi'_{n+l})), \tag{9}$$

$$R'_{pq} = \sum_{m=0}^{M-1-p} \sum_{n=\max(0,-q)}^{\min(N-1-q,\, N-1)} A'_{mn} A'_{(m+p)(n+q)} \exp[j(\varphi'_{mn} - \varphi'_{(m+p)(n+q)})]. \tag{10}$$

To satisfy (7) and (8) for $l = 0$ and $p = q = 0$, the specified amplitudes A_n and A_{mn} should be multiplied by an appropriate constant so that the following energy equivalence to hold:

$$\sum_{n=0}^{N-1} A_n^2 = \sum_{n=0}^{N-1} A_n'^2, \tag{11}$$

$$\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{mn}^2 = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{mn}'^2. \tag{12}$$

Now, (7) and (8) which have some nonlinear equations must be solved to find the unknown phases φ_n and φ_{mn}, respectively. To this end, there are some suitable methods among optimization. Useful error functions can be defined as follows for (7) and (8), respectively,

$$\text{error} = \frac{1}{R'_0} \sqrt{\frac{1}{N} \sum_{l=0}^{N-1} | R_l - R'_l |^2}, \tag{13}$$

$$\text{error} = \frac{1}{R'_{00}} \sqrt{\frac{1}{(2M-1)(2N-1)} \sum_{p=-(M-1)}^{M-1} \sum_{q=-(N-1)}^{N-1} | R_{pq} - R'_{pq} |^2}. \tag{14}$$

V. Examples and Discussion

We consider an example for both linear and planar arrays to investigate the presented method providing initial values of phases by a random generator in the range of $[-\pi, \pi]$.

A. Linear array

A linear array with $N = 21$ antennas is designed for a pencil-beam with identical sidelobes $SLL = -20\,\text{dB}$. The specified amplitude A_n is chosen as symmetric linear variation whose maximum to minimum is equal to 2.5. Figure 1 shows the resultant power pattern of amplitude-phase and phase-only synthesized arrays. Figure 2 compares the autocorrelation coefficients of these two types of arrays. The value of error defined in (13) is just 0.036. The autocorrelation coefficients of desired pencil-beam pattern are real numbers because the power pattern is symmetric. Figure 3 illustrates the amplitude of the excitations for both amplitude phase and phase-only patterns. Moreover, Figure 4 shows the required phase of the phase-only pattern while the required phases of amplitude-phase pattern are zero.

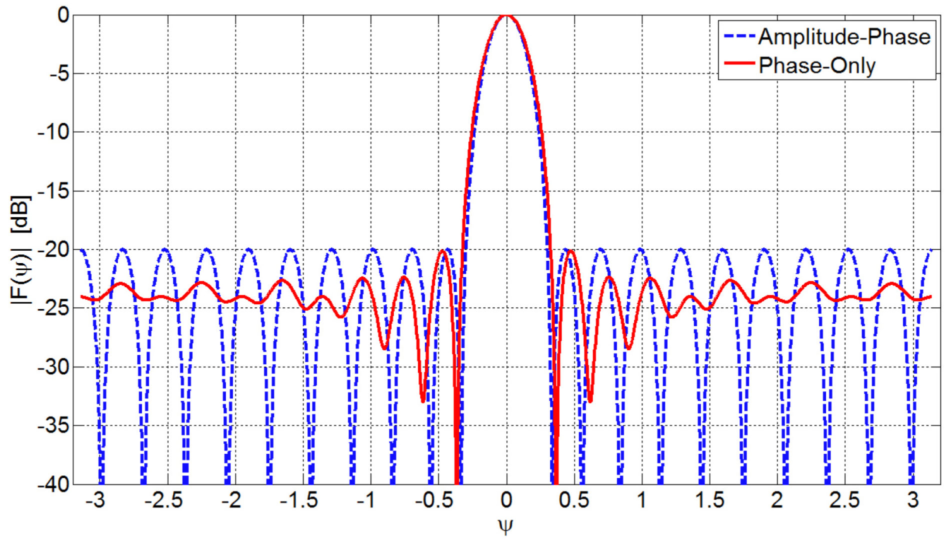

Figure 1. Power pattern of amplitude-phase and phase-only synthesized arrays to have pencil-beam pattern of $SLL = -20$ dB.

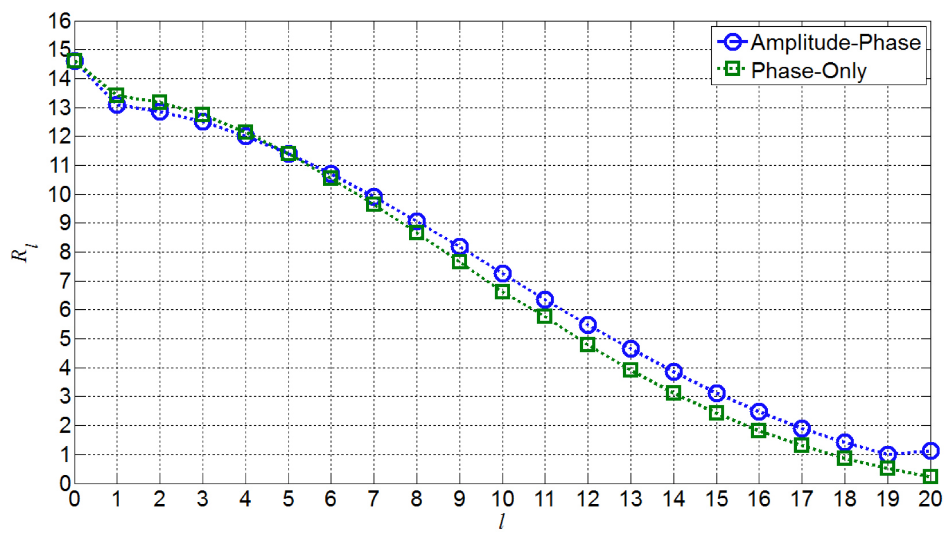

Figure 2. Autocorrelation coefficients of amplitude-phase and phase-only pencil-beam patterns (error = 0.036).

It is seen that the phase-only synthesized power pattern obtained by AMM is satisfactory. This is because the autocorrelation coefficients of the phase-only arrays are close to those of amplitude-phase arrays. It is notable that AMM like other phase-only synthesis methods yields only desired power pattern. Therefore, the phase of synthesized radiation pattern of phase-amplitude and phase-only arrays are not necessarily close to each other. Figure 5 confirms this fact showing a large difference between the phases of two synthesized patterns.

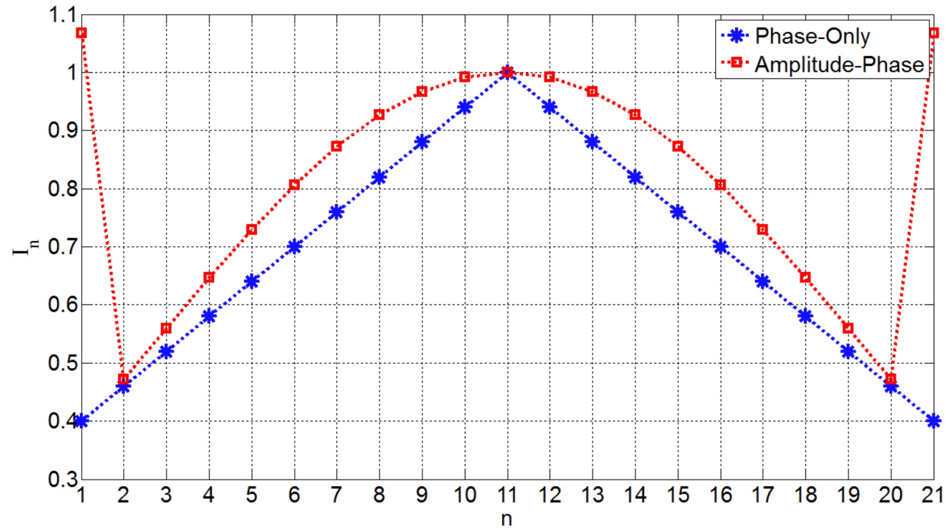

Figure 3. Amplitude of the excitations for both amplitude-phase and phase-only pencil-beam patterns.

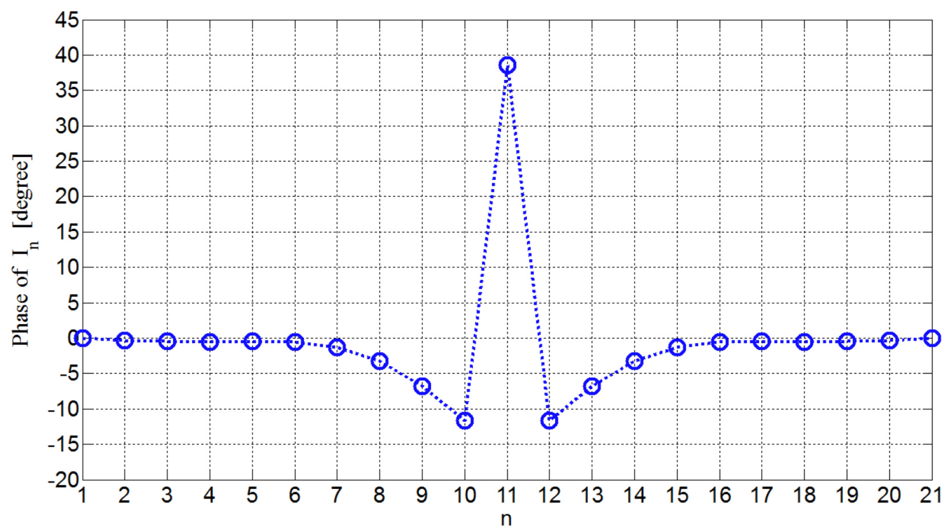

Figure 4. Required phase of antennas for phase-only pencil-beam patterns.

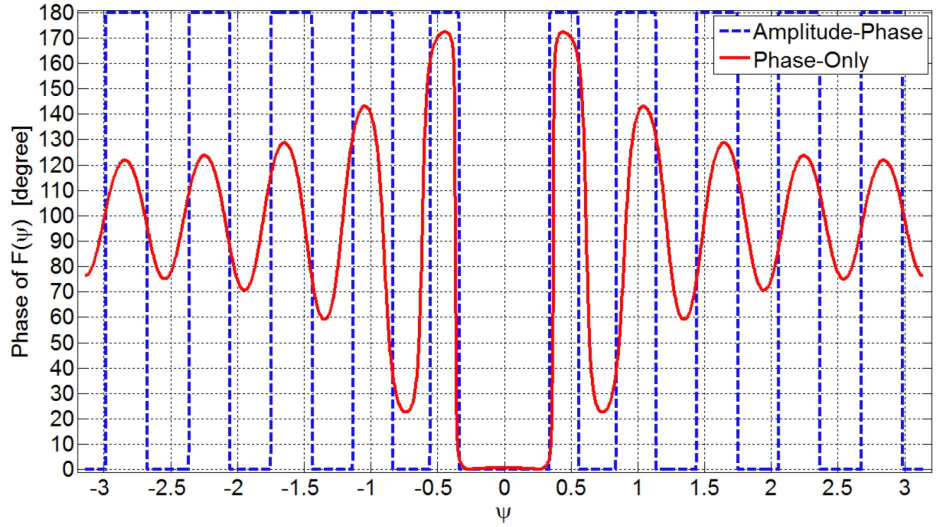

Figure 5. Phase of radiation pattern of amplitude-phase and phase-only synthesized arrays to have pencil-beam pattern of $SLL = -20$ dB.

B. Planar array

A planar array with $M \times N = 21 \times 21$ antennas is designed to have a pencil-beam along with sidelobes of the same level of –20dB. This planar array can be synthesized as the product of two similar linear arrays previously designed. The specified amplitudes A_{mn}, the required phases of the phase-only synthesis, and the resultant power pattern of phase-only synthesized array are shown in Figures 6-8, respectively.

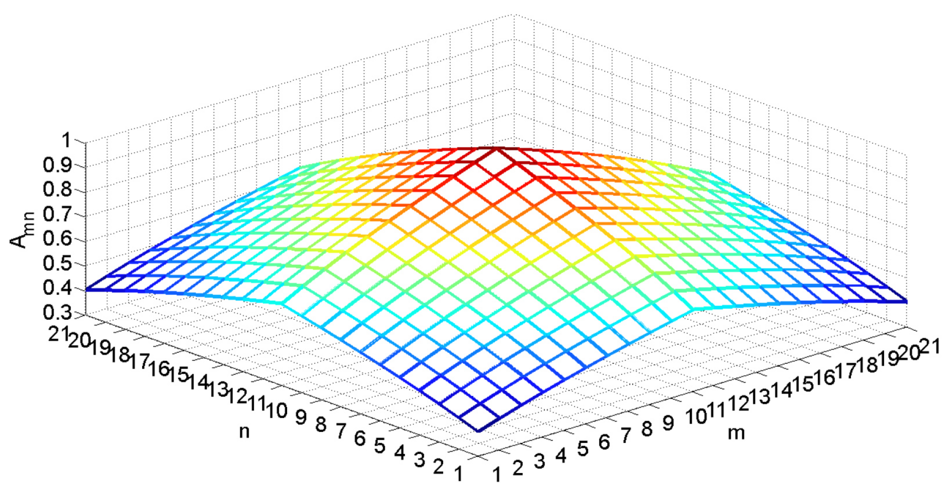

Figure 6. Specified amplitudes of the excitations for phase-only synthesis.

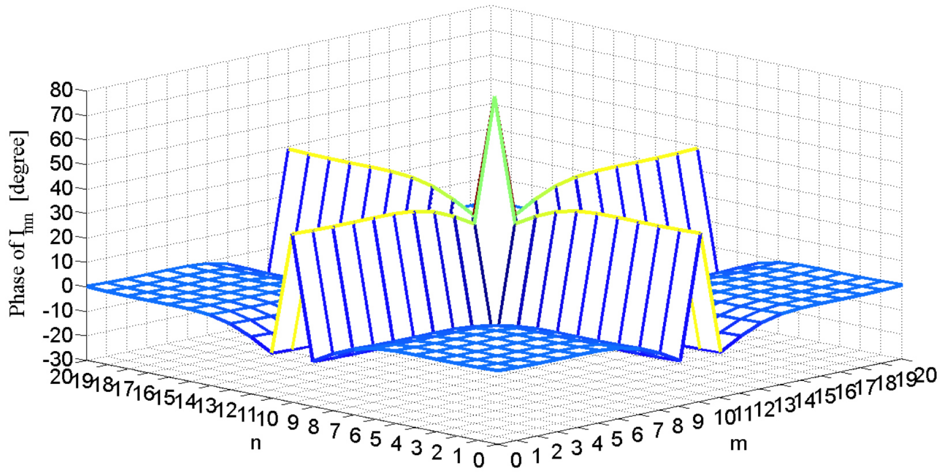

Figure 7. Required phase of antennas for phase-only pencil-beam patterns utilizing the product of two linear arrays.

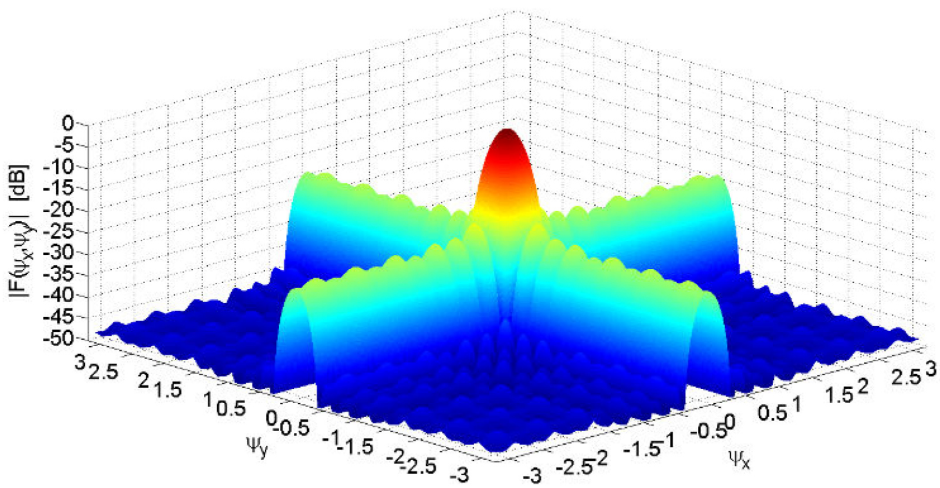

Figure 8. Power pattern of phase-only synthesized planar array utilizing the product of two linear arrays.

Besides, the sidelobes of desired array to be ring-type can be considered, in which amplitude-phase array has to be synthesized making use of the transformation method [3, 4] on the linear array designed previously. Figure 9 shows the power pattern of the amplitude-phase synthesized pattern, while Figure 10 depicts the required amplitudes of amplitude-phase array whose required phases φ'_{mn} are zero. Figure 11 illustrates the autocorrelation coefficients of desired pencil-beam pattern which is real due to its symmetry.

The desired pencil-beam pattern is phase-only synthesized using AMM. The amplitude of the antennas is uniform given as $A_{mn} = 1$. The synthesized phases are shown in Figure 12. It is seen that the distribution of phases is not so regular. Figure 13 shows the phase-only synthesized power pattern. Figure 14 shows the real part of autocorrelation coefficients of phase-only synthesized pattern analogous to Figure 11. The imaginary part of autocorrelation coefficients though not zero is negligible. The value of error defined in (14) is just 0.0084. On comparison of Figure 13 with Figure 9, the good performance of the proposed AMM can be seen.

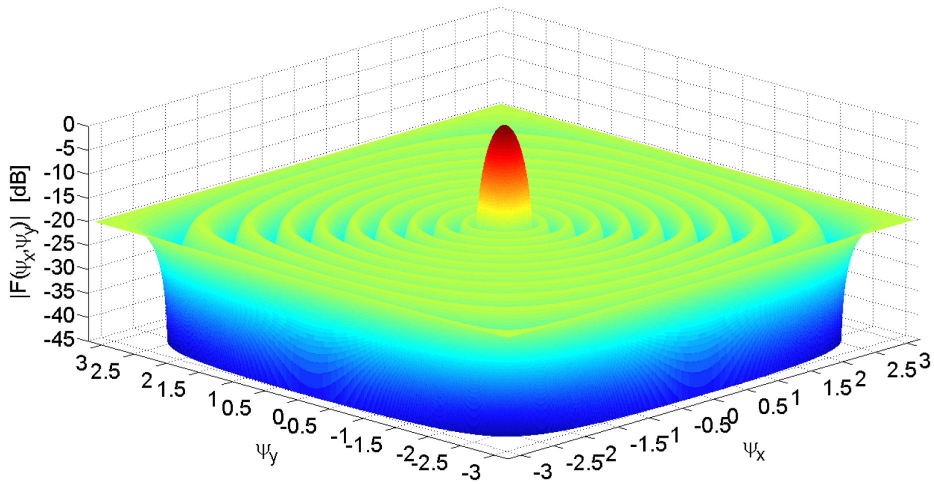

Figure 9. Power pattern of amplitude-phase synthesized planar array using a transformation on a linear array.

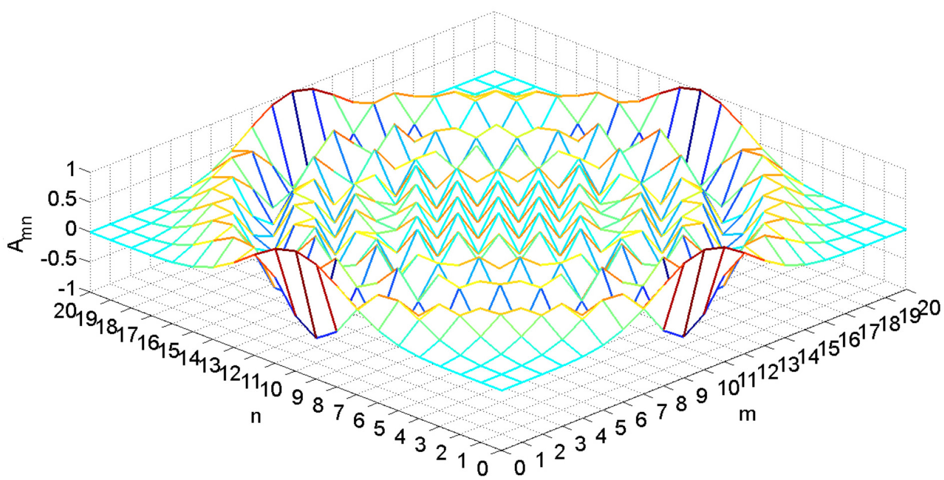

Figure 10. Amplitude of the excitation currents obtained by amplitude-phase synthesis. All phases of excitation currents are zero.

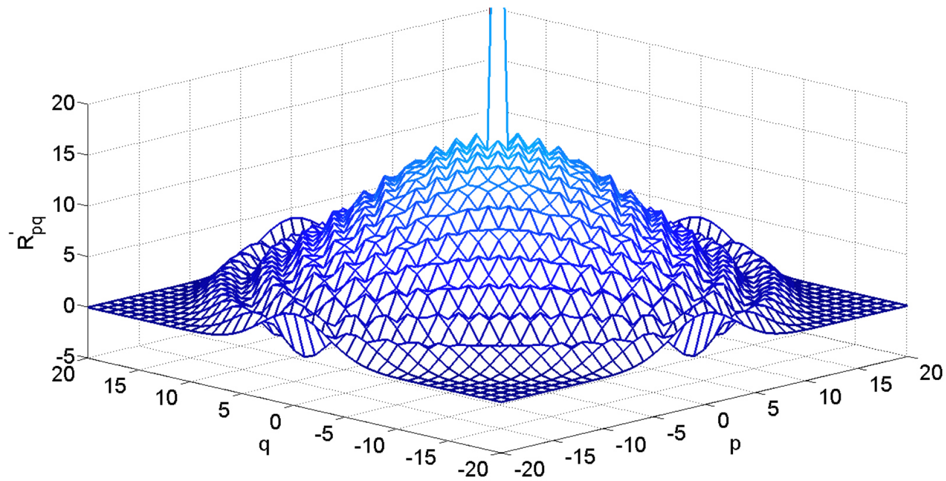

Figure 11. Autocorrelation coefficients of the amplitude-phase synthesized pattern, $R'_{00} = 58.1$.

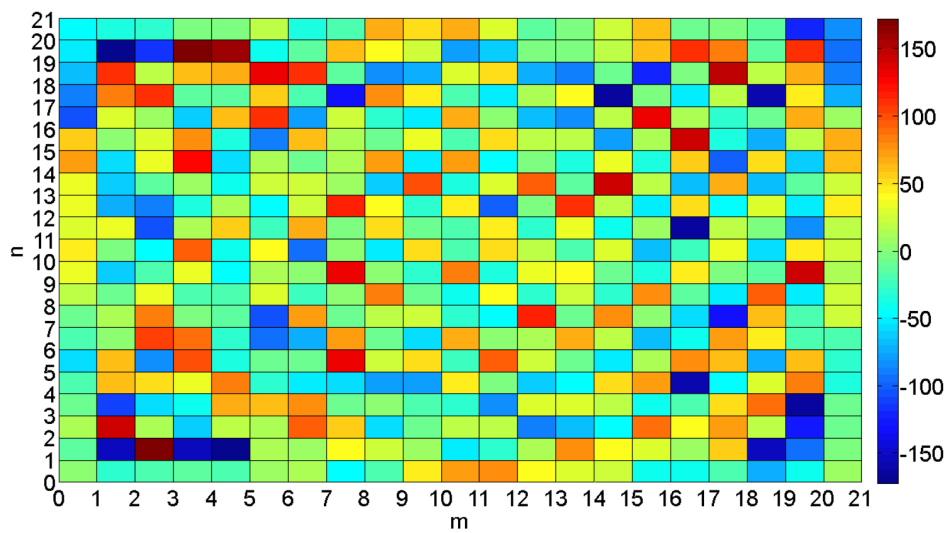

Figure 12. Phases of the excitation currents for pencil-beam pattern obtained by phase-only synthesis. The amplitudes of excitation currents are uniform.

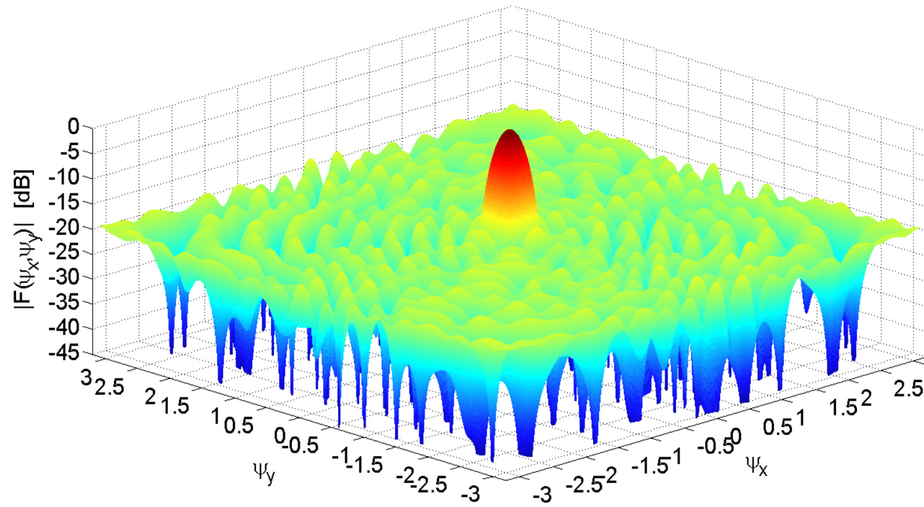

Figure 13. Phase-only synthesized pencil-beam pattern having symmetric sidelobes obtained by AMM method.

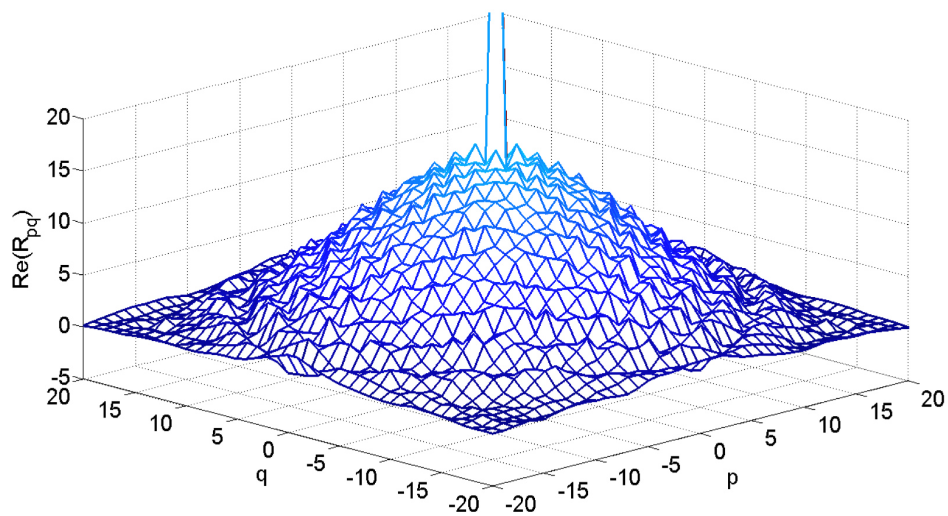

Figure 14. Autocorrelation coefficients of the phase-only synthesized pencil-beam pattern, $R_{00} = 58.1$.

VI. Conclusion

Autocorrelation Matching Method (AMM) was proposed for phase-only synthesis of linear and planar antenna arrays, wherein the autocorrelation coefficients of an array having prefixed amplitudes are equated to those of a previously designed amplitude-phase array. Its effectiveness for pencil-beam pattern of linear and planar arrays is also shown.

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