FAR EAST JOURNAL OF ELECTRONICS AND COMMUNICATIONS
https://sci.pphmjopenaccess.com/index.php/fjec
Volume 30, Number 1, 2026, Pages 21-28
https://dx.doi.org/10.17654/0973700626003
P-ISSN 0973-7006

OPTIMUM SYNTHESIS OF NONUNIFORMLY EXCITED AND UNEQUALLY SPACED ANTENNA ARRAYS

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Abstract

Nonuniformly Excited and Unequally Spaced Arrays (NEUSA) are synthesized optimally. These general type of linear antenna arrays are synthesized to have maximum directivity and minimum Amplitude Dynamic Range (ADR) for a specified sidelobe level (SLL). For this purpose, both positions of antennas and the excitation currents of antennas are considered variable and are optimized simultaneously. A relation is obtained for calculating directivity of NEUSAs versus their amplitude and positions. The presented method is investigated by a comprehensive example. The optimum synthesized arrays have both higher directivity and lower amplitude dynamic range than both equally spaced arrays and uniformly excited arrays.

I. Introduction

Synthesis of antenna arrays with specified sidelobe levels and maximum directivity is important for many applications such as communication and radar systems [1]. The directivity, sidelobe level (SLL) and amplitude dynamic range (ADR) are three important characteristics of antenna arrays which are dependent on the inter-distances between the antennas and the excitation currents of the antennas [2-4].

Uniformly excited arrays of inter-distances equal or more than a half wavelength have the maximum possible directivity and the minimum amplitude dynamic range, i.e. ADR = 1.

Keywords and phrases: Nonuniformly Excited and Unequally Spaced Arrays (NEUSA), maximum directivity, specified sidelobe level.

Received: August 28, 2025; **Accepted:** September 4, 2025;

Published: September 15, 2025

However, the sidelobe level of uniformly excited arrays is high and about –13.2 dB. It is favorable to synthesize arrays so that they have maximum directivity and minimum ADR while their sidelobe level is a desired value. In [5], the levels of sidelobes are set to arbitrary values but the resulted directivity is not necessarily maximum as well as ADR is not necessarily minimum. In [6-7], optimization methods are addressed to synthesize arrays of maximum directivity for a specified sidelobe levels or a specified beamwidth. In [8], a method is proposed to control the beamwidth of the arrays although does not present maximum directivity. In [9-10], analytic methods are proposed to design arrays so that they have as low as possible sidelobe level while having directivity as close as to that of uniformly excited arrays.

On the other hand, nonuniformly spaced antenna arrays have are minimum amplitude dynamic range, i.e. $\text{ADR} = 1$. Many methods have been introduced for the synthesis of nonuniformly spaced antenna arrays. Various optimization procedures [11-13], Zeros Matching Method [14-15], Fourier's Coefficients Equating [16-17] and Orthogonal Coefficients Equating Method [18] are some of these methods. The main drawback of nonuniformly spaced antenna arrays is that the sidelobe level could not be imposed as low as desired.

In this article, Nonuniformly Excited and Unequally Spaced Arrays (NEUSA) are synthesized optimally. In fact, both positions and excitation currents of linear antenna arrays are optimized, simultaneously. We synthesize NEUSAs, optimally so that they have maximum possible directivity and minimum possible ADR for a specified sidelobe level. To this end, a relation is obtained to relate the directivity to the positions and excitation currents of antennas and then both optimum positions and optimum excitation currents are determined. Optimizing both position and excitation currents of antennas, yields higher directivity and lower ADR than ordinary arrays.

The paper is organized as follows: In Section II, NEUSAs and their directivity are introduced and optimally synthesizes. In Section III, some examples are presented and discussed to investigate the performance of the presented method.

II. Analysis and Synthesis of Arrays

Figure 1 shows typical symmetric linear nonuniformly spaced antenna arrays having $L = 2N + 1$ elements. The position of the *n*-th element is as follows:

$$x_n = (n + e_n)d_0; \; n = -N, ..., 0, ..., N, \tag{1}$$

where d_0 is an arbitrary distance close to half of a wavelength, λ, and $e_n s$ are defined as deviations, assuming $e_{-n} = -e_n$.

The array factor or radiation pattern of nonuniform arrays, assuming symmetric excitation could be written as follows:

$$F(\psi_0)=\sum_{n=0}^{N} A_n \cos(kx_n\cos\theta+\alpha_n)=\sum_{n=0}^{N} A_n \cos((n+e_n)\psi_0). \tag{2}$$

In Eq. (2), $A_0=I_0$ and $A_n=2|I_n|$ for $n\neq 0$, where $I_n=|I_n|\exp(j\alpha_n)=|I_n|\exp(j(n+e_n)\alpha)$ is the excitation current of both n-th and $(-n)$-th elements, where $\alpha_n=-\alpha_{-n}=(n+e_n)\alpha=-kx_n\cos\theta_0=-kd_0(n+e_n)\cos\theta_0$ in which θ_0 is the angle of maximum radiation. Also, $\psi_0=kd_0\cos\theta+\alpha$ is the space parameter, where $k=2\pi/\lambda$ and $\alpha=-kd_0\cos\theta_0$.

Amplitude Dynamic Range (ADR) is defined as $\max(|I_n|)/\min(|I_n|)$ which is equal or larger than one. ADR depicts the range of variation of amplitude of the excitation currents and should be as low as possible and near one.

The directivity of linear antenna arrays of symmetric excitations can be obtained using Eq. (2) in the following relation [2-3].

$$D=\frac{4\pi|F(\theta_0)|^2}{\int_0^{2\pi}\int_0^{\pi/2}|F(\theta)|^2\sin\theta d\theta d\varphi}=\frac{2kd_0|F(0)|^2}{\int_{-kd_0+\alpha}^{kd_0+\alpha}|F(\psi_0)|^2 d\psi}. \tag{3}$$

After some mathematical manipulations, the directivity is obtained as follows:

$$D=\frac{2\left(\sum_{n=0}^{N}A_n\right)^2}{\sum_{n=0}^{N}\sum_{m=0}^{N}\sum_{s=1}^{2}A_nA_m\left[\operatorname{sinc}\left(\frac{kd}{\pi}((n+(-1)^s m)+(e_n+(-1)^s e_m))\right)\cos(((n+(-1)^s m)+(e_n+(-1)^s e_m))\alpha)\right]}. \tag{4}$$

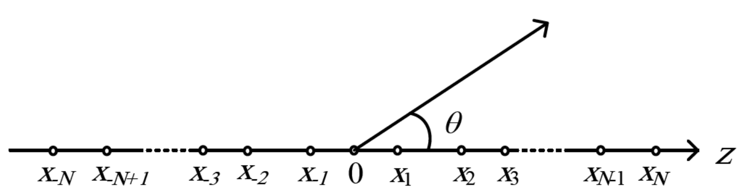

Figure 1. Typical configuration of symmetric linear nonuniformly spaced antenna arrays.

To optimally synthesize the aforesaid antenna array, an error function is proposed and is to be minimized. The proposed error function has three terms to maximize directivity, to minimize ADR and to lead SLL toward SLL_{max}.

$$\text{error} = \frac{1}{D} + \left| \frac{\text{SLL}}{\text{SLL}_{\max}} - 1 \right| + \frac{ADR - 1}{2N + 1}. \tag{5}$$

The proposed error function in Eq. (5), can be made minimum by suitable adopting both the position deviations, e_n, and amplitudes, A_n. This error function must be minimized subject to two following constraints:

(a) The first constraint is making the largest sidelobe level less than a specified value, i.e. $\text{SLL} < \text{SLL}_{\max}$.

(b) The second constraint is making the distances between all two adjacent antennas more than a specified value, i.e. $x_{n+1} - x_n > d_{\min}$. This constraint is to avoid serious coupling between two adjacent elements.

To design such optimum arrays, any appropriate constrained minimization method can be used. Here, we have used the "trust-region-reflective" constrained optimization algorithm which is available in MATLAB environment as *fmincon* function.

III. Examples and Discussion

To verify the proposed method to optimally synthesize antenna arrays, a comprehensive example is presented. A linear array with $L = 2N + 1 = 11$ antenna is designed to have maximum-directivity subject to $\text{SLL}_{\max} = -20\,\text{dB}, -30\,\text{dB}$ and –40 dB with inter-distances more than $d_{\min} = 0.4\lambda$, assuming broadside radiation, i.e. $\theta_0 = 90^\circ$. The optimization is done in three following cases:

A. Uniformly excited array

The amplitudes of excitation currents are considered the same, and only the positions are considered variable. In this case, the first constraint, i.e. $\text{SLL} < \text{SLL}_{\max}$, is not applicable. Figure 2 shows the optimum deviations for this case. Also, Figure 3 compare the resultant radiation pattern and directivity of this case with those of well-known uniformly excited and equally spaced array. It is seen that SLL decreases from –13.2 dB to –16.4 dB at the cost of slightly reducing directivity from 11 to 10.8.

B. Equally spaced array

The inter-distances are considered the same as $d_0 = 0.5\lambda$ and only the amplitudes are considered variable. Figure 4 shows the optimum amplitudes for $\text{SLL}_{\max} = -20\,\text{dB}$, –30 dB and –40 dB. The optimum resultant ADRs are 1.68, 4.03 and 8.70, respectively. It is seen that ADR is increased by decreasing $\text{SLL}_{\max}$. Also, Figures 5-7 illustrate the resultant radiation patterns and directivities of for $\text{SLL}_{\max} = -20\,\text{dB}$, –30 dB and –40 dB, respectively. The optimum resultant directivities are 10.56, 9.30 and 8.32, respectively. It is seen that directivity is decreased by decreasing SLL_{max}.

C. Nonuniformly excited and unequally spaced array (NEUSA)

Both amplitude of excitation currents and inter-distances are considered variable. Figures 2 and 4 show the optimum deviations and optimum amplitudes, respectively, for $SLL_{max} = -20$ dB, –30 dB and –40 dB. The optimum resultant ADRs are 1.01, 2.17 and 5.16, respectively. It is seen that ADRs are significantly less than those of equally spaced arrays. Also, Figures 5-7 illustrate the resultant radiation patterns and directivities of for $SLL_{max} = -20$ dB, –30 dB and –40 dB, respectively. It is seen that directivities are somewhat more than those of equally spaced arrays.

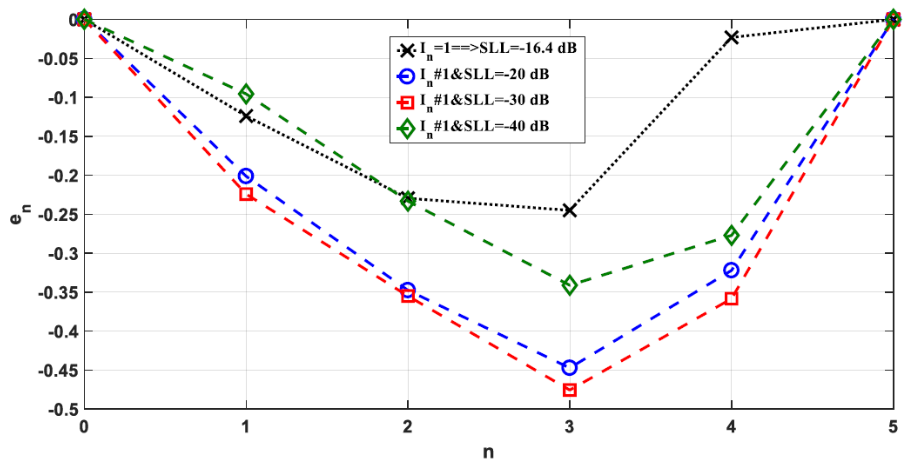

Figure 2. Optimum deviations in uniformly excited array and nonuniformly excited and unequally spaced array.

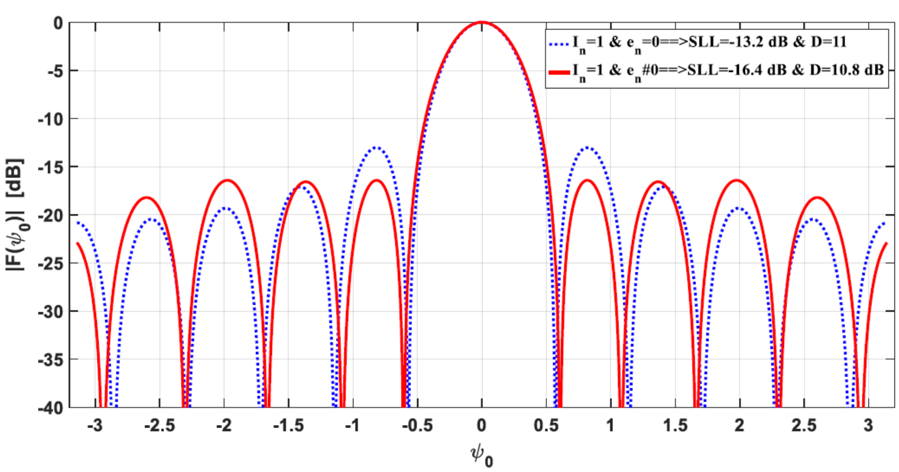

Figure 3. Radiation pattern and directivity for uniformly excited array and uniformly excited and equally spaced array.

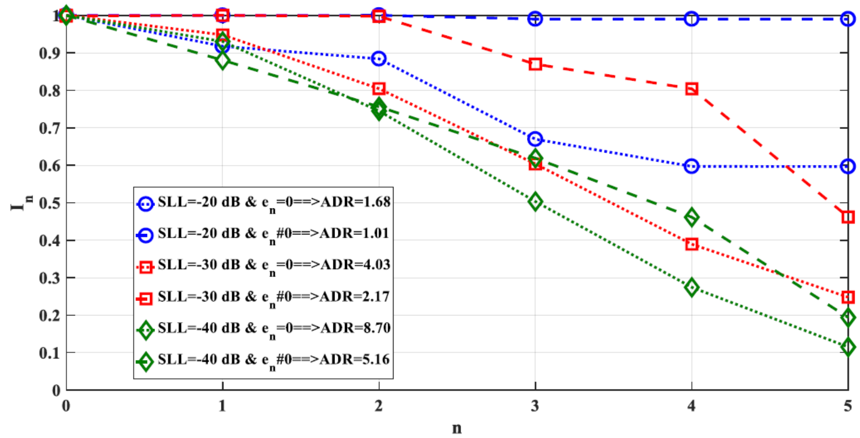

Figure 4. Optimum amplitudes in equally spaced array and nonuniformly excited and unequally spaced array (NEUSA).

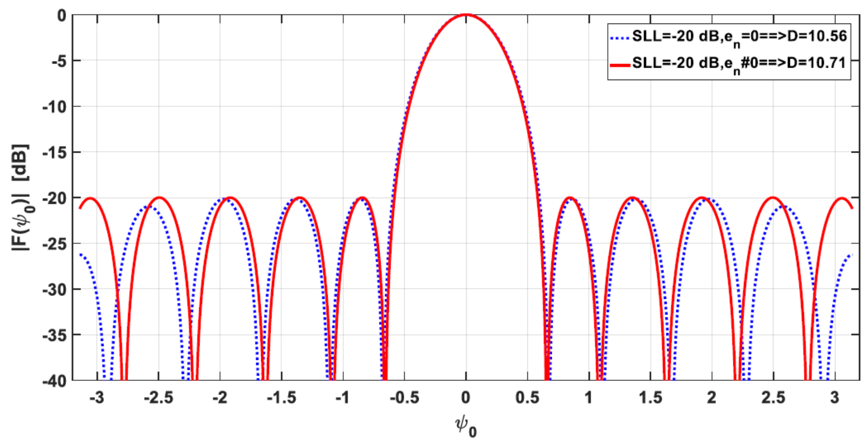

Figure 5. Radiation pattern and directivity for equally spaced array and nonuniformly excited and unequally spaced array (NEUSA), for $SLL_{max} = -20$ dB.

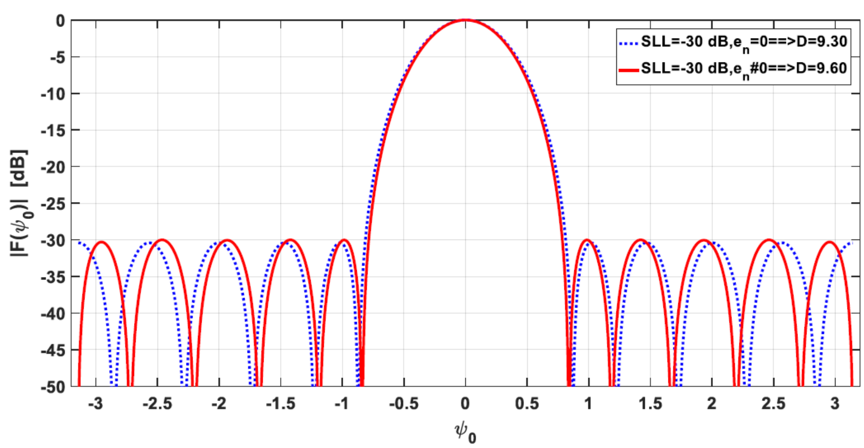

Figure 6. Radiation pattern and directivity for equally spaced array and nonuniformly excited and unequally spaced array (NEUSA), for $SLL_{max} = -30$ dB.

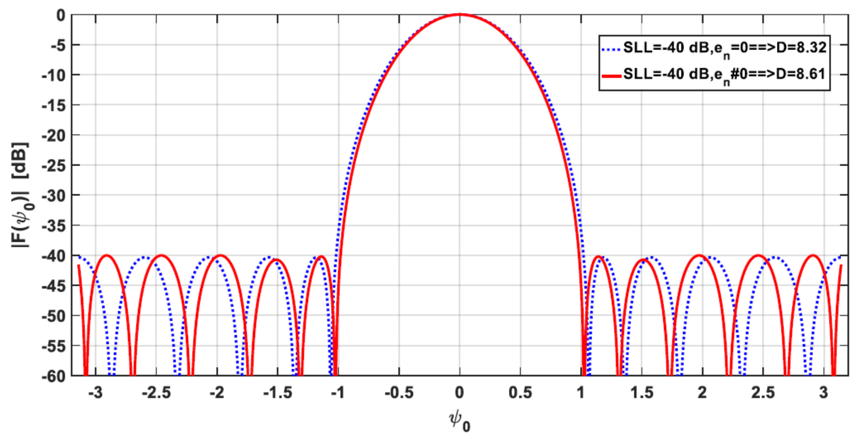

Figure 7. Radiation pattern and directivity for equally spaced array and nonuniformly excited and unequally spaced array (NEUSA), for $\mathrm{SLL}_{\max} = -40$ dB.

IV. Conclusion

Nonuniformly Excited and Unequally Spaced Arrays (NEUSA) were introduced and then synthesized optimally. These general types of linear antenna arrays are synthesized to have maximum directivity and minimum Amplitude Dynamic Range (ADR) for a specified sidelobe level (SLL). For this purpose, both positions of antennas and the excitation currents of antennas are considered variable and are optimized simultaneously. A relation is obtained for calculating directivity of NEUSAs versus their amplitude and positions. The presented method is investigated by a comprehensive example. The optimum synthesized arrays have both higher directivity and lower amplitude dynamic range than both equally spaced arrays and uniformly excited arrays.

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